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## LUBRICATION OF ROLLING ELEMENT BEARINGS

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### ABSTRACT

E-370      This paper is a broad survey of the lubrication of rolling-element bearings. Emphasis is on the critical design aspects related to speed, temperature, and ambient pressure environment. Types of lubrication including grease, jets, mist, wick, and through-the-race are discussed. The paper covers the historical development, present state of technology, and the future problems of rolling-element bearing lubrication.

### INTRODUCTION

In this survey of the lubrication of rolling-element bearings, all the common methods of lubrication are discussed. For the great bulk of bearings in industrial machinery, well defined procedures for designing satisfactory lubrication systems are available in manufacturers' catalogs and standard handbooks. The discussions on these more common methods are rather brief in this paper. The aircraft gas turbine engine has provided the impetus and driving force for quantum leaps in lubrication technology. For that reason, this paper leans heavily toward discussions of that technology.

### FUNCTIONS OF A LIQUID LUBRICANT

A liquid or grease lubricant in a rolling-element bearing provides several functions. A major function is to separate the surfaces of the raceways and the rolling elements with an elastohydrodynamic (EHD) film. The formation of the EHD film depends on the elastic deformation of the contacting surfaces and the hydrodynamic properties of the lubricant. The magnitude of the EHD film is dependent mainly on the viscosity of the lubricant and the speed and load conditions on the bearing. For normal bearing geometries, the magnitude of the EHD film thickness is of the order of 0.1 to 1.0 micron (4 to 40  $\mu$ in.).

It was only about 30 years ago that it was recognized that a lubricant in the form of an EHD film could separate heavily loaded contacts in rolling-element bearings. Grubin [1] in 1949 developed an approximate film thickness equation for EHD line contacts. It was the mid-1950's before the first experimental evidence of an EHD film was obtained. In 1958 Crook [2]

<sup>1</sup>Numbers in brackets denote references at end of paper.

reported on EHD film thickness measurements by capacitance and oil flow techniques which tended to confirm earlier theoretical predictions. Since that time, a multitude of variations and improvements of EHD theory as well as film thickness measurements have been published.

In many applications, conditions are such that total separation of the surfaces is not attained. That is, some contact of asperities occurs. Since the surfaces of the raceways are not ideally smooth and perfect, the existing asperities may have greater height than the generated EHD film and penetrate the film to contact the opposing surface. When this happens, it is a second function of the lubricant to prevent or minimize surface damage from this contact. Action of additives in the lubricants aid in protecting the surfaces by reacting with the surfaces and forming films which prevent gross damage. Contacts between the cage and the rolling-elements and the cage and guiding lands on the race may also be lubricated by this means.

If the operating conditions are such that the asperity contacts are frequent and sustained, significant surface damage can occur when the lubricant can no longer provide sufficient protection. The lubricant film parameter  $\Lambda$  is a measure of the adequacy of the lubricant film to separate bearing surfaces.  $\Lambda$  is the ratio of the EHD film thickness to the composite surface roughness  $\sigma$ , where  $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$  and  $\sigma_1$  and  $\sigma_2$  are the RMS roughness of the two surfaces in contact. In order for the frequency of asperity contacts between the rolling surfaces to be negligible,  $\Lambda$  must be greater than 3. When  $\Lambda$  is much less than 1, one should expect significant surface damage and short bearing life. When  $\Lambda$  is between approximately 1.5 and 3, some asperity contact occurs, but satisfactory bearing operation and life can be obtained due to the protection provided by the lubricant. Further discussion on failure modes and effects of  $\Lambda$  on bearing life are contained in a later section of this paper.

Predicting the range of  $\Lambda$  for a given application is dependent on knowing the magnitude of the EHD film thickness to a fair degree of accuracy. RMS surface roughness can, of course, be measured but may be modified somewhat during run-in. The EHD film thickness can be calculated by one of several equations. An easily applied equation developed by Hamrock and Dowson [3], for either point or line contacts is

$$H_{min} = 3.63 U^{0.68} G^{0.49} W_p^{-0.073} (1 - e^{-0.68k}) \quad (1)$$

where  $H_{min}$  is the dimensionless minimum film thickness and  $k$  is a simplified expression for the ellipticity parameter

$$k = 1.03 \left( \frac{R_y}{R_x} \right)^{0.64} \quad (2)$$

The other terms are defined in the appendix.

Convenient relations for determining the effective radii of curvature and surface velocities are given by Anderson in [4] for both ball and cylindrical roller bearings.

Liquid lubricants also serve other functions in rolling-element bearings. Heat generated in a bearing can be removed if the lubricant is circulated through the bearing either to an external heat exchanger or simply brought into contact with the system casing or housing. Other cooling techniques with recirculating lubricant systems will be discussed later. Circulating lubricant also flushes out wear debris from intermittent contact in the bearing. Both liquid lubricants and greases can act as rust and corrosion preventatives and help to seal out dirt, dust, and moisture.

#### SOLID FILM LUBRICATION

When operation of rolling-element bearings is required at extreme temperatures, either very high or very low, or at low pressures (vacuum), normal liquid lubricants or greases are not usually suitable. High temperature limits are due to thermal or oxidative instability of the lubricant. At low

temperatures, such as in cryogenic systems, the lubricants' viscosity is so high that pumping losses and bearing torque are unacceptably high. In high vacuum systems or space applications, rapid evaporation limits the usefulness of liquid lubricants and greases.

For these unusual environmental conditions, rolling-element bearings can be lubricated by solid films. The use of solid film lubrication generally limits bearing life to considerably less than the full fatigue life potential available with proper oil lubrication. Solid lubricants may be used as bonded films, transfer films or loose powder applications.

Transfer film lubrication is employed in cryogenic systems such as rocket engine turbopumps. The cage of the ball or roller bearing is typically fabricated from a polytetrafluoroethylene (PTFE) containing material [5]. Lubricating films are formed in the raceway contacts by PTFE transferred from the balls or rollers which have rubbed the cage pocket surfaces and picked up a film of PTFE. Cooling of bearings in these applications is readily accomplished since they are usually operating in the cryogenic working fluid. In cryogenic applications where radiation may also be present, PTFE filled materials are not suitable, but lead and lead alloy coated cages can supply satisfactory transfer film lubrication [6].

In very high temperature applications, lubrication with loose powders or bonded films, has provided some degree of success. Powders such as molybdenum disulphide, lead monoxide, and graphite have been tested up to 922 K (1200° F) [7]. However, neither loose powders nor bonded films have seen much use in high temperature rolling-element bearing lubrication. Primary use of bonded films and composites containing solid film lubricants occurs in plain bearings and bushings in the aerospace industry. Further discussions of solid film lubrication can be found in [8 and 9].

#### GREASE LUBRICATION

Perhaps the most common, widely used, most simple, and most inexpensive mode of lubrication for rolling-element bearings is grease lubrication. Lubricating greases consist of a fluid phase of either a petroleum oil or a synthetic oil and a thickener. Additives similar to those in oils are used, but generally in larger quantities.

The lubricating process of a grease in a rolling-element bearing has been described in the literature in a way that is perhaps oversimplified. The thickener phase acts essentially as a sponge or reservoir to hold the lubricating fluid. In an operating rolling-element bearing, the grease generally channels or is moved out of the path of the rolling balls or rollers and a portion of the fluid phase bleeds into the raceways and provides the lubricating function.

Scarlett [10], attempted to observe the mechanism of grease lubrication. He found that the fluid in the contact areas of the balls or rollers and the raceways appeared to be grease in which the thickener had broken down in structure due to its being severely worked. This fluid did not resemble the lubricating fluid above. Also, as shown by Dyson and Wilson [11], the EHD film thickness with grease does not react to changes in speed as would be expected from the lubricating fluid alone, which indicates some more complicated lubricating mechanism with grease. It appears that more work must be done to understand and be able to predict the EHD film forming nature of grease.

Grease lubrication is generally used in the more moderate rolling-element bearing applications, although some of the more recent grease compositions are finding use in severe aerospace environments such as high temperature and the vacuum of space. The major advantages of a grease lubricated rolling-element bearing are simplicity of design, ease of maintenance, and minimized weight and space requirements.

Greases are retained within the bearing, thus they do not remove wear debris and degradation products from the bearing. The grease is retained either by shields or seals or by the design of the housing. Positive contact seals can add to the heat generated in the bearing. Greases do not remove heat from a bearing as a circulating liquid lubrication system does.

The speed limitations of grease lubricated bearings are due mainly to limited capacity to dissipate heat, but are also affected by bearing type and cage type. Standard quality (ABEC-1 or 3) ball and cylindrical roller bearings with stamped steel cages are generally limited to 0.2 to 0.3 million DN. (DN is a speed parameter which is the bore in millimeters multiplied by the speed in a rpm). More precision bearings (ABEC-6 or 7) with machined metallic or phenolic cages may be operated as high as 0.4 to 0.6 million DN. Grease lubricated tapered roller bearings and spherical roller bearings are generally limited to less than 0.2 million DN and 0.1 million DN, respectively. These limits are basically those stated in bearing manufacturers' catalogs. Additional discussion of uses and limitations of grease lubrication in rolling-element bearings is found in [10].

The selection of type or classification of grease, by both consistency and type of thickener, is based on the temperatures, speeds, and pressures to which the bearings are to be exposed. A comprehensive discussion of grease types and compositions can be found in reference [12]. For most applications, the rolling-element bearing manufacturer can recommend the type of grease and in some cases can supply bearings prelubricated with the recommended grease.

Although in many cases a piece of equipment with grease lubricated ball or roller bearings may be described as "sealed for life" or "lubricated for life," it should not be assumed that grease lubricated bearings have infinite grease life. It may only imply that that piece of equipment has a useful life less than that of the grease lubricated bearing. On the contrary, grease in an operating rolling-element bearing has a finite life which may be less than the calculated fatigue life of the bearing. Grease life is limited by evaporation, degradation, and/or leakage of the fluid from the grease. To preclude bearing failure due to inadequate lubrication or worn out grease, periodic relubrication should be scheduled.

The period of relubrication is generally based on experience with known or similar systems. An equation estimating grease life in ball bearings has been published by Booser [13]. The basis for the equation is a compilation of life tests on many sizes of bearings in electric motors. Factors in the equation account for type of grease, size of bearing, temperature, speed, and load. With a combination of operating experience and trends from the Booser equation, a satisfactory relubrication schedule can generally be applied. When possible it would be wise to observe relubrication intervals recommended by equipment manufacturers, since relubrication should be based on representative grease life experience.

#### JET LUBRICATION

For rolling-element bearing applications where speeds are too high for grease or simple splash lubrication, jet lubrication is frequently used to lubricate and control bearing temperature by removing generated heat. In jet lubrication, the placement of the nozzles, number of nozzles, jet velocity, lubricant flow rates, and removal of lubricant from the bearing and immediate vicinity are all very important for satisfactory operation. Even the internal bearing design is a factor to be considered.

The importance of careful and proper jet lubricating system design was shown by Matt and Giannotti [14] and is summarized in Fig. 1. For seemingly identical bearing operating conditions of load, speed and lubricant flow rates, drastic difference in bearing operation and temperature can be seen. In the case shown by the dashed lines, temperature rises drastically with increased flow rate due to use of a simple oil jet directed at the loaded side of the ball bearing inner ring and inadequate scavenge port size. For the case shown by the solid lines, properly placed multiple jets and larger scavenge ports with an external scavenge pump allowed a decrease in temperature with increased lubricant flow rate to an optimum level. It is obvious that some care must be taken in designing a jet lubricated bearing system.

The proper placement of jets should take advantage of any natural pumping ability of the bearings. This is illustrated in Fig. 2 for a ball bearing with relieved rings and for a tapered roller bearing. Centrifugal forces aid in moving the oil through the bearing to cool and lubricate the elements.

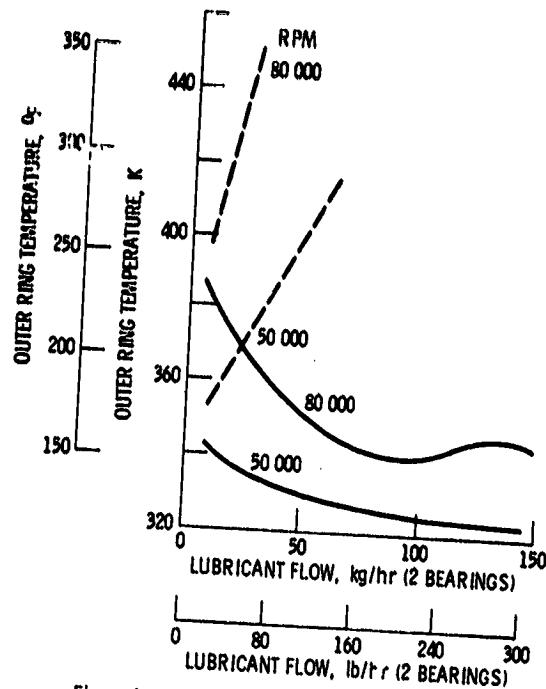


Figure 1. - Effectiveness of proper jet lubrication.  
Solid line denotes multiple, well placed jet nozzle  
with adequate scavenge. Dashed line denotes single  
jet with inadequate scavenge. (Test bearings, 20 mm  
bore angular contact ball bearings; thrust load, 222 N  
(50 lb); from [14].)

Directing jets in the radial gaps between the rings and the cage is beneficial. The design of the cage and the lubrication of its surfaces sliding on the rings greatly effects the high speed performance of jet lubricated bearings. The cage has been typically the first element to fail in a high-speed bearing with improper lubrication.

It is shown in [15 and 16] that with jet lubrication outer-ring-riding cages give lower bearing temperatures and allow higher speed capability than inner-ring-riding cages. It is expected that with an outer-ring-riding cage, where the larger radial gap is between the inner ring and the cage, better penetration and thus better cooling of the bearing is obtained. Some conflicting data exists, however, for 120-mm- (4.7244-in.-) bore ball bearings [17] which shows cooler bearing temperature with an inner-ring-riding cage. This difference has not been resolved.

Lubricant jet velocity is, of course, dependent on flow rate and nozzle size. Miyakawa [1b], in experiments with 30-mm- (1.1811-in.-) bore deep groove ball bearings with outer-ring-riding cages, showed that jet velocity has significant effects on bearing temperature. Figure 3 from [15] shows that jet velocities greater than 10 m/sec (33 ft/sec) and preferably near 20 m/sec (66 ft/sec) are required for good penetration and cooling of the bearing. The penetration ratio is the ratio of lubricant flow transmitted through the bearing to the total lubricant flow from the jet nozzle.

It has been shown [15] that with proper bearing and cage design, placement of nozzles, jet velocities and adequate scavenge, jet lubrication can be successfully used for small bore ball bearings to speeds up to 3.0 million DN. Likewise for large bore ball bearings [17], speeds to 2.5 million DN are attainable. For large bore tapered roller bearings (120.65-mm (4.75-in.) bore), jet lubrication was successfully demonstrated to 1.8 million DN [18], although a

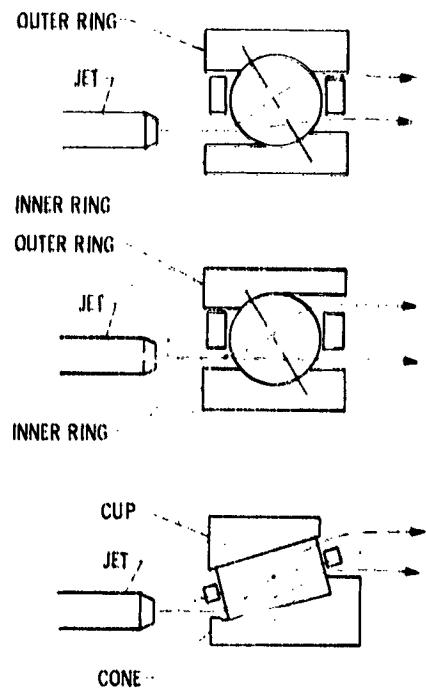


Figure 2. - Placement of jets for ball bearings with relieved rings and tapered-roller bearings.

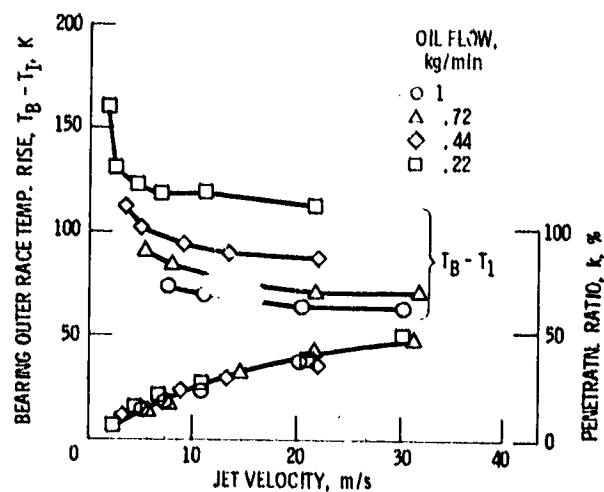
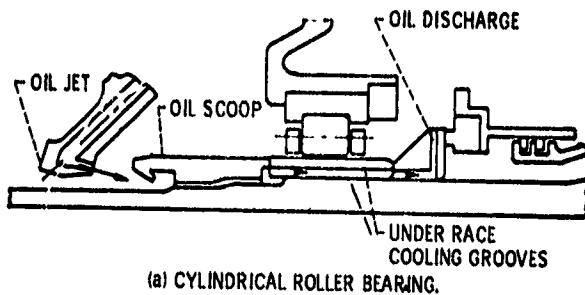


Figure 3. - Bearing outer race temperature rise and penetration ratio versus jet velocity. (Test bearing, 6206 (outer race-riding cage); shaft speed, 60 000 rpm; thrust load, 50 kg; inlet oil temperature,  $T_1$ , 303 K; from [15].)

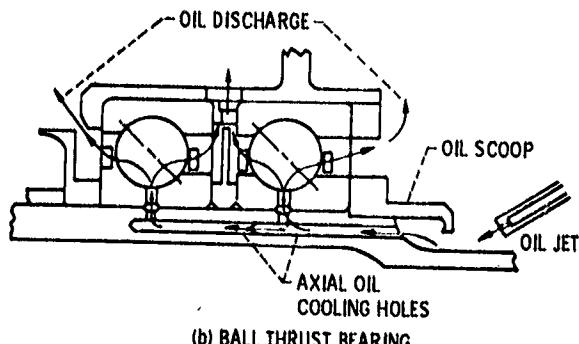
high lubricant flow rate of 0.0161 m<sup>3</sup>/min (4.0 gpm) and a relatively low oil-in temperature of 360 K (170° F) were required.

#### UNDER-RACE LUBRICATION

During the mid-1960's, as speeds of the main shaft of turbojet engines were pushed upwards, a more effective and efficient means of lubricating rolling-element bearings was developed. Conventional jet lubrication failed to adequately cool and lubricate the inner race contact as the lubricant was thrown centrifugally outward. Increased flow rates only added to heat generation from churning the oil. Brown [19] described an "underrace oiling system" used in a turbofan engine for both ball and cylindrical roller bearings. Figure 4, from [19], shows the technique used to direct the lubricant under and centrifugally out through holes in the inner race to cool and lubricate the bearing. Some lubricant may pass completely through under the bearing for cooling only as shown in Fig. 4(a). Although not shown in the figure, some radial holes may be used to supply lubricant to the cage riding lands.



(a) CYLINDRICAL ROLLER BEARING.



(b) BALL THRUST BEARING.

Figure 4. - Underrace oiling system for main shaft bearings on turbofan engine. (From [19].)

This lubricating technique has been thoroughly tested for large bore ball and roller bearings up to 3 million DN. Results of these tests have been published by Holmes [20] with 125-mm- (4.9212-in.-) bore ball bearings, Signer, et al. [21] with 120-mm- (4.7244-in.-) bore ball bearings, Brown, et al. [22] with 124-mm- (4.8819-in.-) bore cylindrical roller bearings, and Schuller [23] with 118-mm- (4.6457-in.-) bore cylindrical roller bearings. An example of the effectiveness of under-race lubrication and cooling is shown in Fig. 5 from [17]. Under-race lubricated ball bearings ran significantly cooler than identical bearings run with jet lubrication. Beyond 16 700 rpm (2 million DN), the bearing temperature with under-race lubrication increased only nominally while that with jet lubrication increased at an accelerated rate. Only at reduced load could the jet lubricated bearings be run at 2.5 million DN.

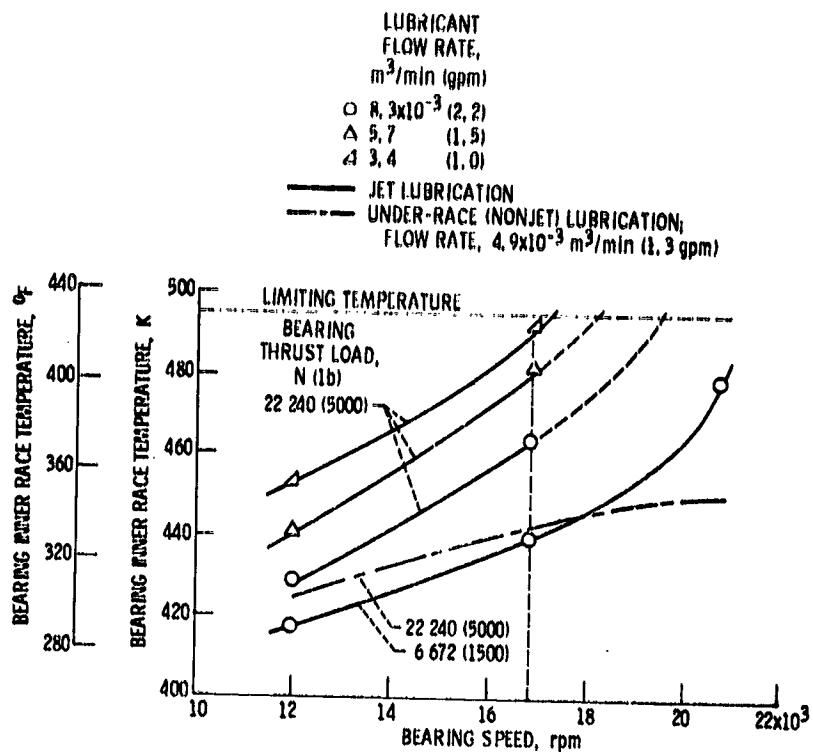


Figure 5. - Effectiveness of under-race lubrication with 120 mm-bore angular contact ball bearings. (Oil-in temperature, 394 K (250° F); from [17].)

Under-race lubrication was successfully used under a variety of load conditions up to 3 million DN [17 and 21].

Applying under-race lubrication to small bore bearings (<40-mm bore) is more difficult because of limited space available for grooves and radial holes, and means to get the lubricant under the race. For a given DN value, centrifugal effects are more severe with small bearings since centrifugal forces vary with DN<sup>2</sup>. Heat generated per unit of surface area is also much higher, and heat removal is more difficult in smaller bearings.

Although it was previously shown that operations up to 3 million DN can be successfully achieved with small bore bearings with jet lubrication, some advantages may be attained if under-race lubrication can be used. Schuller [24] has shown significantly cooler inner-race temperatures with 35-mm- (1.3780-in.-) bore ball bearings with under-race lubrication. As shown in Fig. 6, from [24], the effect is greater at higher speeds up to 72 000 rpm (2.5 million DN).

Tapered roller bearings have been restricted to lower speed applications relative to ball and cylindrical roller bearings. The speed limitation is primarily due to the cone-rib/roller-end contact which requires very careful lubrication and cooling consideration at higher speeds. The speed of tapered-roller bearings is limited to that which results in a DN value of approximately 0.5 million DN (a cone-rib tangential velocity of approximately 36 m/sec (7000 ft/min) unless special attention is given to lubricating and designing this cone-rib/roller-end contact. At higher speeds, centrifugal effects starve this critical contact of lubricant.

In the late 1960's, the technique of under-race lubrication was applied to tapered-roller bearings, that is, to lubricate and cool the critical cone-rib/roller-end contact. As described in [25], 88.9-mm- (3.5-in.-) bore tapered roller bearings were run under combined radial and thrust loads to 1.42 million DN with cone-rib lubrication (the term used to denote under-race lubrication in tapered-roller bearings).

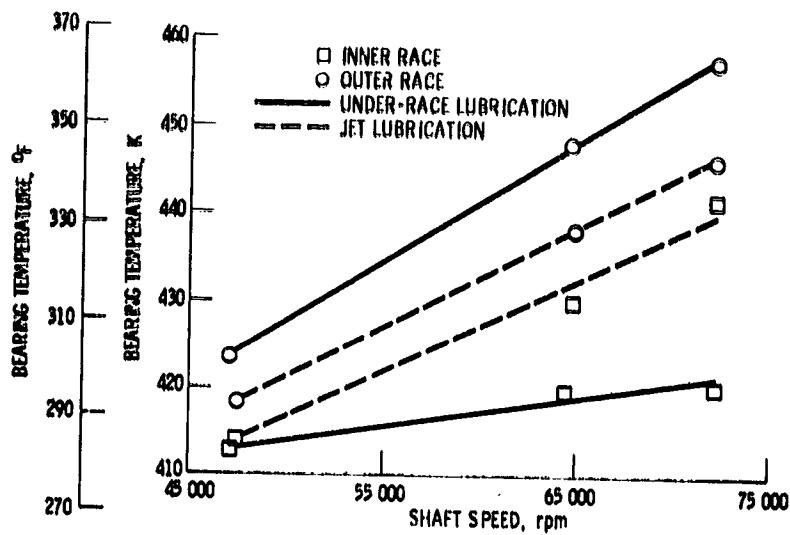


Figure 6. - Effect of under-race lubrication with 35-mm bore angular contact ball bearings. (Total oil flow rate,  $1318 \text{ cm}^3/\text{min}$  (0.348 gpm); oil-in temperature, 394 K ( $250^\circ \text{ F}$ ); from [24].)

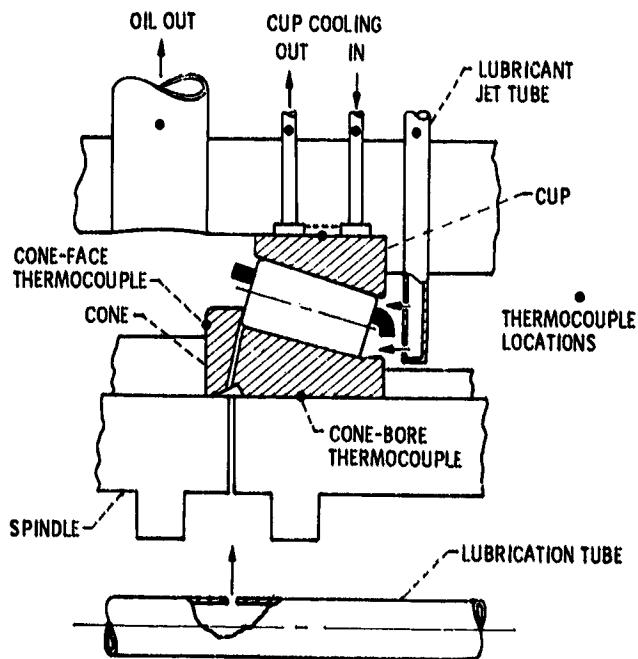


Figure 7. - Tapered-roller bearing with cone-rib and jet lubrication.  
(From [18].)

A comparison of cone-rib lubrication and jet lubrication was reported in [18] for 120.65-mm- (4.75-in.-) bore tapered-roller bearings under combined radial and thrust loads. These bearings were standard catalog bearing design except for the large end of the roller which was made spherical for a more favorable contact with the cone-rib. Those bearings that used cone-rib lubrication also had holes drilled through from a manifold in the cone bore to the undercut at the large end of the cone, as shown in Fig. 7. The results of [18] show very significant advantage of cone-rib lubrication as shown in Fig. 8. At 15 000 rpm (1.8 million DN) the bearing with cone-rib lubrication had a cone face temperature 34 K ( $64^{\circ}$  F) lower than one with jet lubrication. Furthermore, [18] shows that the tapered-roller bearing would operate with cone-rib lubrication at 15 000 rpm with less than half the flow rate required for jet lubrication at that speed.

Further work has shown successful operation with large bore tapered-roller bearings at even higher speeds. Orvos [26] reported on long term operation of 107.95-mm- (4.25-in.-) bore tapered roller bearings under pure thrust load to 3 million DN with a combination of cone-rib lubrication and jet lubrication. Parker, et al., [27] showed successful operation of optimized design 120.65-mm- (4.75-in.-) bore tapered-roller bearings under combined radial and thrust load with under-race lubrication to both large end (cone-rib) and small end to speeds up to 2.4 million DN.

Under-race lubrication has been shown to very successfully reduce inner-race temperatures. However, at the same time, outer race temperatures either remain high [18] or are higher than those with jet lubrication as shown in Fig. 6 from [24]. The use of outer-race cooling can be used to reduce the outer-race temperature to levels at or near the inner-race temperature. This would further add to the speed capability of under-race lubricated bearings and avoid large differentials in bearing temperature that could cause excessive internal clearance.

The effect of outer-race cooling (or cup cooling in the case of tapered-roller bearings, see Fig. 7) is shown in Table I from [27]. The cup outer-surface temperature is decreased to the cone bore temperature with cup

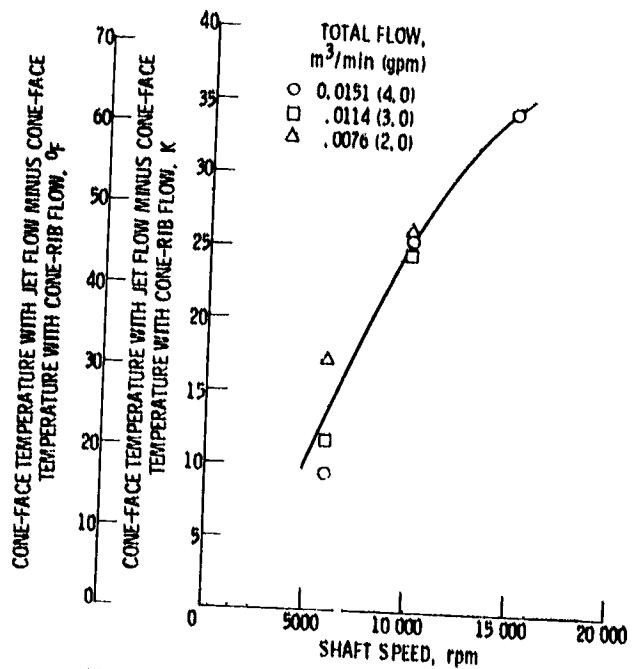


Figure 8. - Advantage of cone-rib lubrication over jet lubrication with 4.75-inch bore tapered-roller bearings. (Oil-in temperature, 350 K ( $170^{\circ}$  F); from [18].)

TABLE 1. - EFFECT OF CUP COOLING ON TAPERED-ROLLER BEARING TEMPERATURES

(Shaft speed, 18 600 rpm; oil-in temperature, 364 K (195° F); total flow rate without cup cooling, 0.0114 m<sup>3</sup>/min (3.0 gpm); from [27].)

Cup cooling flow rate, m <sup>3</sup> /min (gpm)	Temperature, K (°F)				
	Cone face	Cone bore	Cup outer surface	Oil-out	Cup cooling oil-out
0	389 (240)	423 (302)	438 (329)	426 (307)	-----
0.0038 (1.0)	391 (245)	424 (303)	424 (304)	426 (308)	386 (235)

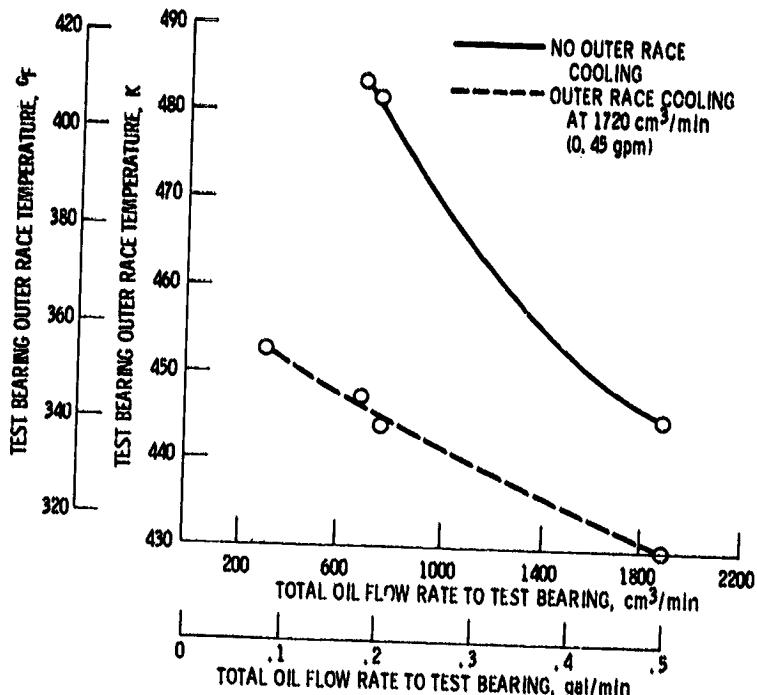


Figure 9. - Effect of outer race cooling on outer race temperature of 35-mm bore ball bearings at 72 300 rpm. (Oil-in temperature, 394 K (250° F); from [24].)

cooling. With the 35-mm- (1.3780-in.-) bore ball bearings of [24], outer-race cooling significantly decreased outer-race temperatures as shown in Fig. 9.

Under-race lubrication has been well developed for larger bore bearings and is currently being used with many aircraft turbine engine mainshaft bearings. Because of the added difficulty of applying it, the use of under-race lubrication with small bore bearings has been minimal, but the benefits have been demonstrated. It appears that application of tapered roller bearings at higher speeds using cone-rib lubrication is imminent, but the experience to date has been primarily in laboratory test rigs.

The use of under-race lubrication in all the previous work referenced, includes the use of holes through the rotating inner race. It must be recognized that these holes weaken the inner-race structure and could contribute to the possibility of inner race fracture at extremely high speeds. This subject of fracture of inner races is discussed by Bamberger [28] in another paper at this conference. It is apparent however, that the fracture problem exists even without the lubrication holes in the inner races.

#### SPLASH OR BATH LUBRICATION

Lubrication of rolling-element bearings with either jets or under-race techniques requires the use of a recirculating oil system. These systems can include external pumps, filters, heat exchangers for oil cooling, scavenging pumps, and flow control devices, or they may be simple built-in pumps merely to circulate lubricant within a gearbox. For many applications where shaft speeds are low and cooling requirements are minimal, less sophisticated methods of lubrication are not only adequate but desirable as well. One such method is splash or bath lubrication where part of the bearing or an adjacent component is immersed in lubricant. This method is generally inexpensive to incorporate and has low maintenance costs. It generally requires little attention and is reliable. Since a finite, and generally small quantity of lubricant is used, the system should be enclosed and well sealed. This is also necessary to keep foreign debris and dirt out of the system since the lubricant does not pass through filters as it would in a circulating system. The technique of splash or bath lubrication is fairly common in industrial machinery where speed and temperature conditions are not severe. Consequently, little if any new technology has been developed or required in this area.

#### WICK LUBRICATION

A wick lubrication system is also relatively simple and inexpensive to incorporate and maintain and is commonly used in many less severe operating conditions. This method uses a wick of felt or waste packing to link an oil reservoir to the bearing. Oil is transported by capillary action in the wick material. The system can be designed to use lubricant recirculation. This circulation allows some filtering of the lubricant.

Some more sophisticated applications where wick lubrication is being used or proposed are in air turbine drives and in energy storage flywheels. In both applications, speeds are generally high, but the environmental temperature is low to moderate. Thus, cooling of the bearings with jet or under-race lubrication is not required.

The flywheel application presents an added environmental condition; that is, flywheels are generally run in a moderate vacuum to reduce windage losses. Wick lubricated bearings promise to be desirable for this application because of relative simplicity and low torque losses.

Figure 10 depicts the wick lubrication of a ball bearing to support an

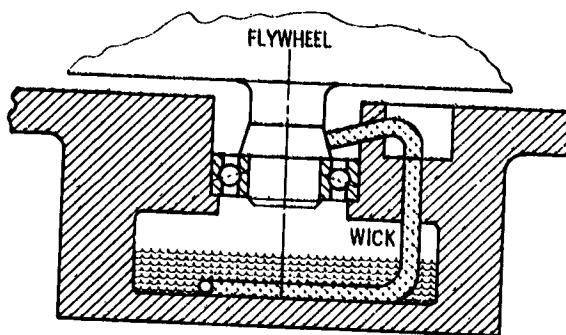


Figure 10. - Wick lubrication of ball bearing for energy storage flywheel,

energy storage flywheel presently undergoing testing at the author's laboratory. The wool felt wick transports oil from the reservoir to a tapered sleeve above the bearing. The oil is centrifugally fed to the bearing, is circulated through the bearing and returned by gravity back to the reservoir. Unpublished tests have shown stable and successful operation with this system with 25-mm bore ball bearings to 20 000 rpm in a vacuum of 0.1 mm Hg.

#### AIR-OIL MIST LUBRICATION

Another commonly used lubrication method for rolling-element bearings is air-oil mist or aerosol lubrication. This method uses a suspension of fine oil particles in air as a fog or mist to transport oil to the bearing. The fog is then reclassified or condensed at the bearing so that the oil particles will wet the bearing surfaces. Reclassification is extremely important, since the small oil particles in the fog do not readily wet bearing surfaces. The reclassifier generally is a nozzle that accelerates the fog, forming larger oil particles that more readily wet bearing surfaces.

Air-oil mist lubrication is non-recirculating; the oil is passed through the bearing once and then discarded. Very low oil flow rates are sufficient for lubrication of rolling-element bearings exclusive of the cooling function. This type lubrication has been used in industrial machinery for over 50 years. It is used very effectively in high-speed, high-precision machine tool spindles.

A recent application of an air-oil mist lubrication system is in an emergency lubrication system for mainshaft bearings in a U.S. Army helicopter turbine engine. An emergency lubrication system was required to allow safe engine operation for up to 30 minutes in the event that the primary lubrication system (recirculating jet) suffered severe ballistic damage.

An experimental program which verifies the feasibility of this approach was reported by Rosenlieb in [29]. It demonstrated that an air-oil mist system along with auxiliary cooling air could provide satisfactory short term operation of a 46-mm bore angular-contact ball bearing to speeds up to 3.0 million DN. Successful tests up to 100 hour duration were performed at 2.0 million DN with a mist oil flow rate of  $284 \text{ cm}^3/\text{hr}$  ( $16.7 \text{ in.}^3/\text{hr}$ ), a cooling air flow rate of  $0.337 \text{ scmm}$  ( $11.9 \text{ scfm}$ ) and a bearing temperature of  $472 \text{ K}$  ( $390^\circ \text{ F}$ ). This oil flow rate to the bearing is at least two orders of magnitude less than would be required in a jet lubrication system for these conditions. However, over a 100

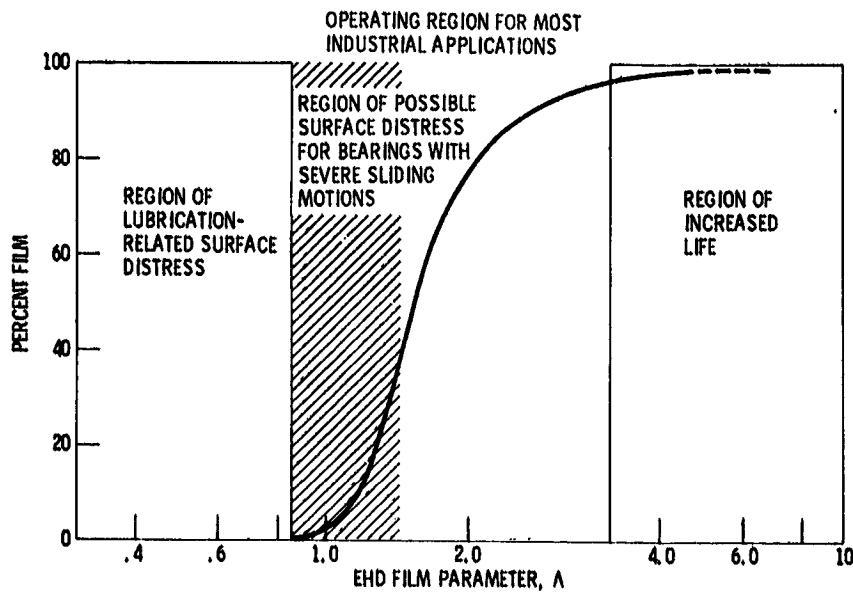


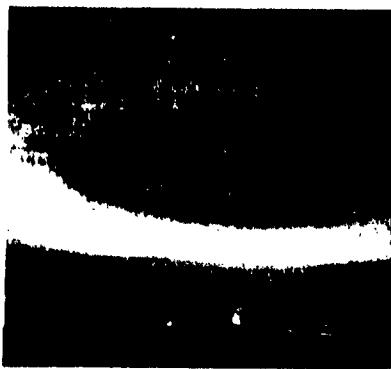
Figure 11. - Percent film as a function of EHD film parameter. (From [31].)

hour period, it represents over 76.5 liters (2 gal.) of oil discarded to the environment. For this reason air-oil systems are not generally used for long term operation in the more severe operating conditions such as those in aircraft turbine main shaft bearings.

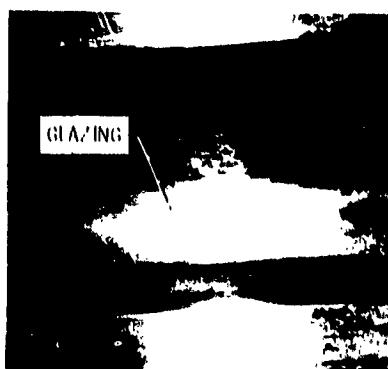
Where air-oil mist systems are more common, such as machine tool spindles, the oil flow rates are significantly less. Systems are commercially available which can be tailored to supply lubricant from a central source for a large number of bearings.

#### LUBRICATION RELATED SURFACE FAILURE MODES

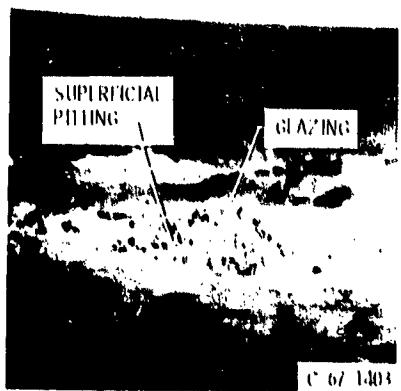
As discussed earlier, the EHD film parameter  $\Lambda$  has a significant effect on whether satisfactory bearing operation is attained. It has been observed that surface failure modes in rolling-element bearings can be generally categorized by the value of  $\Lambda$ . The  $\Lambda$  parameter has been shown to be related to percent film, or the time percentage during which the "contacting" surfaces are fully separated by an oil film, by Tallian [30] and Harris [31]. The relationship is shown in Fig. 11. For  $\Lambda \leq 3$  the surfaces are for practical purposes completely separated.



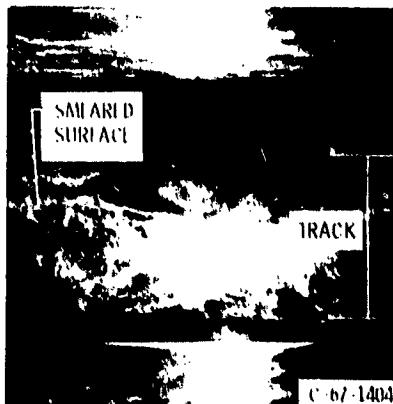
(a) NORMAL RACE APPEARANCE AFTER BEING RUN WITH FULL ELASTOHYDRODYNAMIC LUBRICATION.



(b) RACE APPEARANCE AFTER GLAZING.



(c) RACE APPEARANCE AFTER GLAZING AND SUPERFICIAL PITTING.



(d) RACE APPEARANCE AFTER SMEARING.

Figure 12. Effect of EHD lubrication on surface damage to bearing raceways. (From [33].)

For  $\Lambda < \delta_1$ , various degrees of asperity contact occur. The effect of the degree of contact on surface damage to bearing raceways was related to  $\Lambda$  by Tallian [32] and by Zaretsky and Anderson [33]. Photographs of bearing raceways showing various types of surface damage are shown in Fig. 12. It has been deduced that for  $\Lambda$  less than 1, severe surface distress, such as smearing or deformation with wear will occur as in Fig. 12(d).

The  $\Lambda$  range between 1 and 3 is where many rolling element bearings usually operate. For this range successful operation depends on additional factors such as lubricant/material interactions, lubricant additive effects, the degree of sliding or spinning in the contact, and surface texture (other than rms surface finish). The surfaces may appear as in Fig. 12(b), with only minor surface glazing or deformation of asperity peaks. Or they may appear as in Fig. 12(c), where superficial pitting occurs, and the distress is more severe. This distress generally occurs when there is more sliding or spinning in the contact (such as in angular contact ball bearings) and when the lubricant/material and surface texture effects are less favorable.

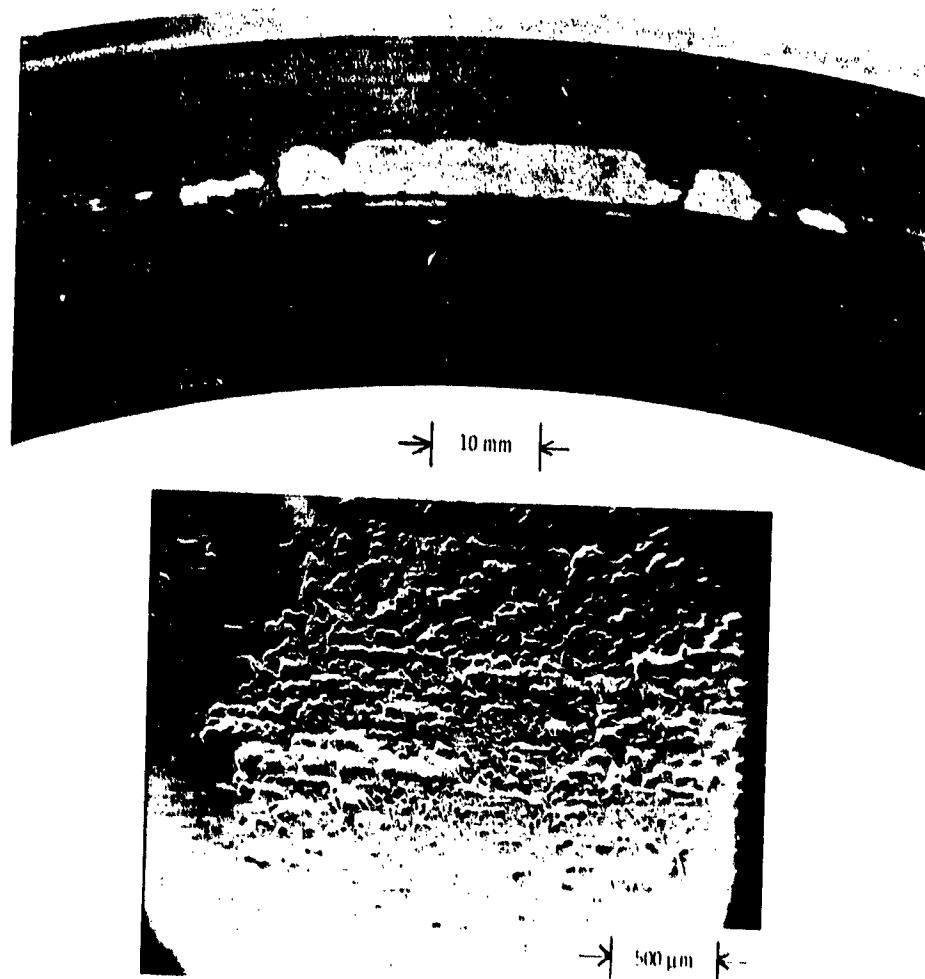


Figure 13. - Peeling on tapered roller bearing raceway.

Another type of surface distress related to film parameter  $A$  is peeling which has been seen in tapered-roller bearing raceways. Peeling, as described by Littmann and Moyer [34], is a very shallow area, uniform in depth usually less than 0.013 mm (0.0005 in.). Fig. 13 shows a cup raceway surface of a tapered-roller bearing with peeling. It was shown that this mode of distress could be eliminated by improving the  $A$  value (unpublished tests at the author's laboratory). By improving surface finish and lowering operating temperature to increase  $A$ , the peeling distress was avoided.

To preclude surface distress and possible early rolling-element bearing failure,  $A$  values less than 3 should be avoided. When this condition is attained, the appearance of the raceway surface should be similar to that of Fig. 12(a) where original grinding marks and texture are unchanged.

#### LUBRICATION EFFECTS ON FATIGUE LIFE

The EHD film parameter  $A$  has added significance with respect to the fatigue life of rolling-element bearings. Tallian [32] related  $A$  to bearing spalling fatigue life as shown in Fig. 14. Note that the curve extends to values less than 1. This implies that even though  $A$  is such that significant surfaces distress could occur, continued operation would result in surface initiated spalling fatigue.

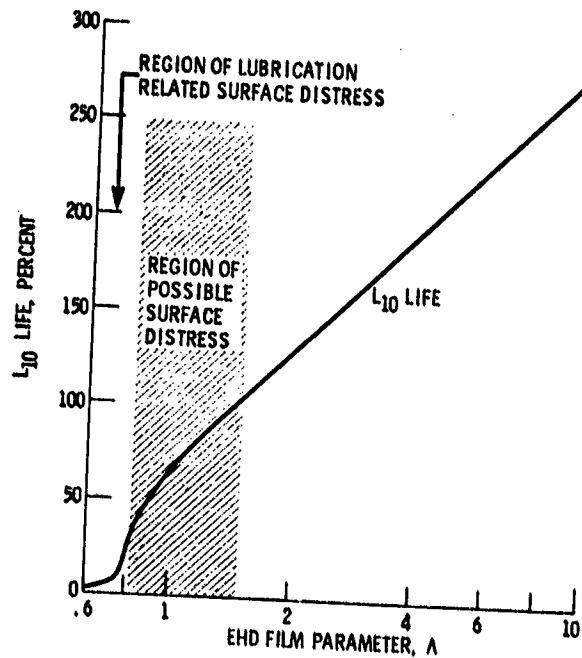


Figure 14. - Group fatigue life  $L_{10}$  as a function of EHD film parameter. (From [32].)

Skurka [35] published results of fatigue tests with cylindrical roller bearings over a range of  $A$  values similar to that of Fig. 14. The authors of [36] have recommended an average curve, Fig. 15, based on both Tallians' [32] and Skurka's [35] data. This curve can be used to determine a lubrication factor. To obtain a bearing life that is adjusted for lubrication effects, the calculated AFBMA bearing life is multiplied by this factor.

The effects of lubrication on fatigue life have been critically explored by Tallian [37] who describes a more sophisticated model which includes asperity slope and traction effects among other factors. This work [37] and that of Liu, et al. [38] is useful in explaining some of the scatter that has been observed in data showing effects of lubrication on fatigue life.

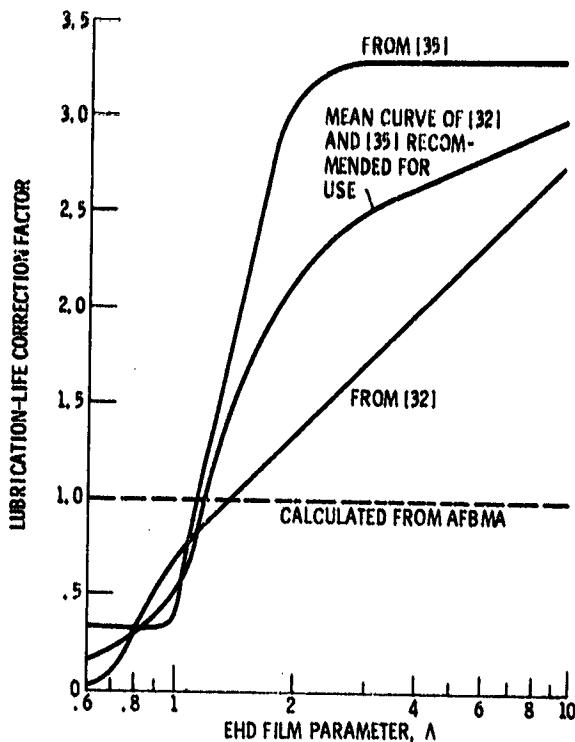


Figure 15. - Lubrication-life correction factor as a function of lambda. (From [36].)

Life correction factors for lubricant effects are now being used in sophisticated computer programs for analysis of rolling-element bearing performance such as those of Crecelius [39] and Kleckner [40]. In such programs, the lubricant film parameter is calculated, and a life correction factor is used in bearing life calculations. Further discussion of these computer codes can be found in another paper at this conference by Pirvics [41].

A practical account of the effect of the lubricant film parameter  $\Lambda$  on bearing fatigue life is given by Russell and Clark [42]. A group of aircraft turbine engine main shaft ball bearings were found to have poor outer raceway surface finishes. The fatigue life determined from rig testing these bearings was found to be significantly less than the life of similar bearings with proper surface finishes and considerably less than predicted by methods of [36]. The bearings with poor outer raceway finishes had a calculated  $\Lambda$  value at the outer-race-ball contact of 0.7. Acceptable finishes on the outer raceway gave a calculated value of approximately 3. The experimental results showed two orders of magnitude life difference, whereas the predicted life difference was considerable less than one order of magnitude.

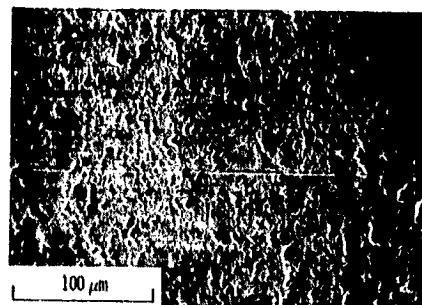
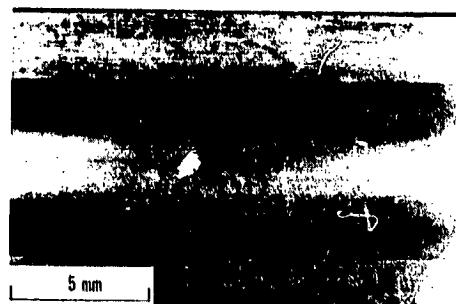
No other published data known to the author have shown such large life difference due to lubrication effects. It is possible that other factors entered into the life reduction such as material/lubricant interactions and chemical effects. Up to now, investigators have concentrated on the physical factors involved to explain the greater scatter in life results at low  $\Lambda$  values. Material/lubricant chemical interactions have not been adequately investigated. From decades of boundary lubrication studies, it is apparent that chemical effects must play a significant role where there is appreciable asperity interaction. More work needs to be done in this area.

#### LUBRICANT CONTAMINATION AND FILTRATION

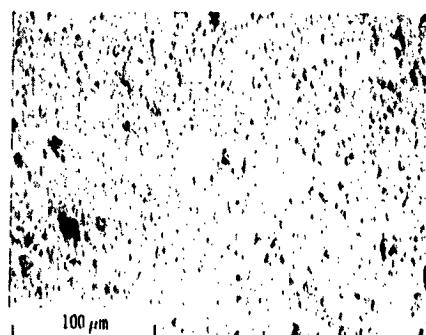
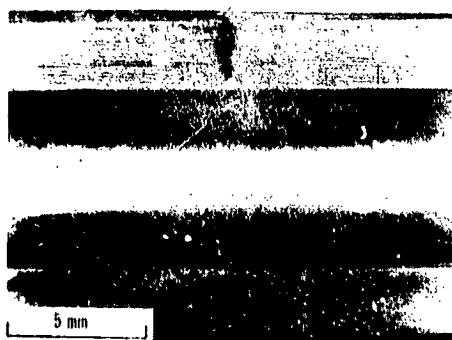
It is well recognized that fatigue failures which occur on rolling-element contacts are a consequence of competitive failure modes developing primarily from either surface or subsurface defects. Subsurface initiated fatigue, that which originates slightly below the surface in a region of high shearing stress, is generally the mode of failure for properly designed, well lubricated, and well maintained rolling-element bearings. Surface initiated fatigue, often originating at the trailing edge of a localized surface defect, is the most prevalent mode of fatigue failure in machinery where strict lubricant cleanliness and/or sufficient EHD film thickness are difficult to maintain.

The presence of contaminants in rolling-element systems will not only increase the likelihood of surface initiated fatigue but can lead to a significant degree of component surface distress. In [43], experiments performed on tapered-roller bearings have shown that wear is proportional to the amount of contamination in the lubricant, and that the wear rate generally increases as the contaminant particle size is increased. Furthermore, the wear process will continue as long as the contaminant particle size exceeds the EHD film separating the bearing surfaces. Since this film thickness is rarely greater than 3 microns (118  $\mu$ in.) for a rolling contact component, even extremely fine contaminant particles can cause some damage.

The research of [44] showed that an 80 to 90 percent reduction in ball bearing fatigue life could occur when contaminant particles were continuously fed into the recirculating lubrication system. Also, ball bearing life tests reported in [45] suggest that the use of an "ultraclean" lubrication system may improve bearing life several-fold over a conventional lubrication system.

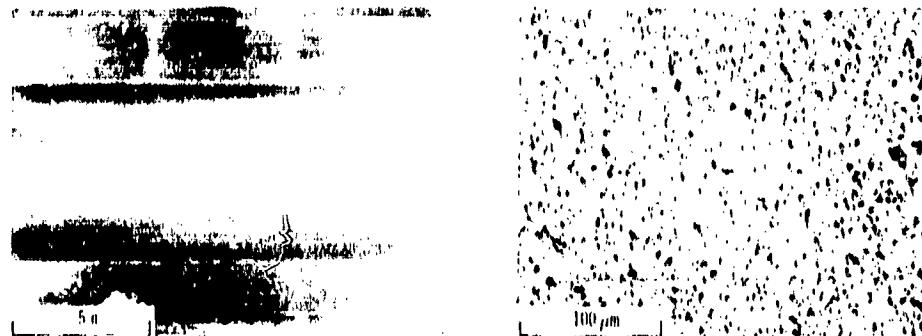


(a) TEST BEARING SUSPENDED AFTER 1172 hours FROM 3-micron-ABSOLUTE FILTER TESTS WITH CONTAMINATED LUBRICANT (TEST SERIES III).

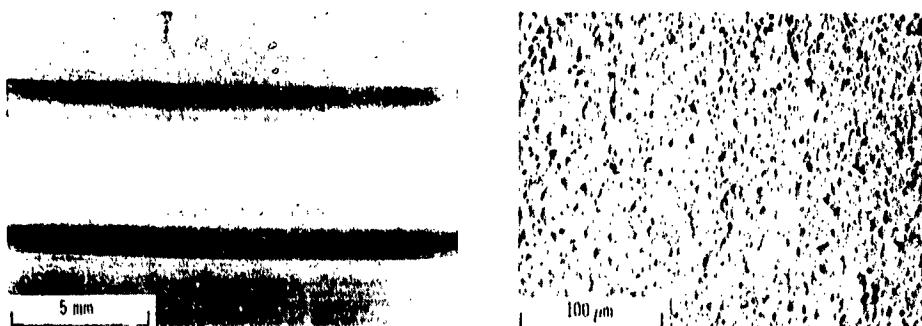


(b) TEST BEARING SUSPENDED AFTER 987 hours FROM 30-micron-ABSOLUTE FILTER TESTS WITH CONTAMINATED LUBRICANT (TEST SERIES III).

Figure 16. - Macro- and SEM-photographs of inner races of 65-mm bore ball bearings showing progressive surface damage of running track with coarser filter size. (From 146.)



(c) TEST BEARING SUSPENDED AFTER 663 hours FROM 40-micron-ABSOLUTE FILTER TESTS WITH CONTAMINATED LUBRICANT (TEST SERIES IV).



(d) TEST BEARING SUSPENDED AFTER 449 hours FROM 105-micron-ABSOLUTE FILTER TESTS WITH CONTAMINATED LUBRICANT (TEST SERIES VI).

Figure 16 . - Concluded.

Loewenthal and Moyer [46] reported that finer filtration improved bearing fatigue life and decreased wear of bearing components run in a system containing lubricant contaminated with carbon, siliceous and metallic particles. The photographs in Fig. 16 show inner raceways of the 66-mm- (2.5590-in.-) bore ball bearings run in these tests and illustrate the progressive increase in surface distress and wear with coarser filter size. This is reflected by an increase in the intensity and width of the wear track coupled with the increasing absence of grinding marks. Although bearing fatigue life was similar for the 3 micron and 30 micron filter tests, the use of filters coarser than 30 microns reduced bearing fatigue life as shown in Table 2.

There has been a reluctance to use fine filters because of the concern that fine lubricant filtration would not sufficiently improve component reliability to justify the possible increase in system cost, weight, and complexity. In addition it is presumed that fine filters will clog more quickly, have a higher clean pressure drop, and generally require more maintenance than currently used filters. The study described by Lynch and Cooper [47] demonstrated that these presumptions are not always correct. In this study, tests were performed on a 3-micron absolute main oil filter which replaced the original production 40-micron nominal (66-micron absolute) filter for a helicopter gas turbine engine lubrication system. The new filter elements provided a much cleaner lubricant with less component wear, while greatly extending the time between filter removals for clogging and oil changes. This was accomplished with a modest increase in filter size and weight and with a new filter clean pressure drop nearly the same as the original production unit. The turboshaft engines which power advanced helicopters such as the Army's UTTAS and AAH now use 3-micron absolute filtration in their lubrication systems.

TABLE 2. - FATIGUE-LIFE RESULTS OF 65 MILLIMETERS BORE BALL BEARING TESTS FOR  
VARIOUS LEVELS OF FILTRATION IN A CONTAMINATED LUBRICANT

(Radial load, 4600 N (1030 lbf); speed, 15 000 rpm; temperature, 347 K  
(106° F); test lubricant, MIL-L-23699 type I from (46).)

Test series <sup>a</sup>	Test filter absolute rating, $\mu$	Experimental hours		Weibull slope	Failure index <sup>b</sup>	Confidence number, <sup>c</sup> percent	
		10-percent life, h <sub>10</sub>	60-percent life, h <sub>60</sub>			10-percent life	60-percent life
I	49	672	2276	1.64	9 out of 22	--	--
II	3	606	993	2.78	10 out of 16	63	99
III	30	694	857	6.12	11 out of 16	67	99
IV	49	367	633	6.06	20 out of 32	89	99
V	106	---	---	----	-----	--	--

<sup>a</sup>Test series I used clean oil, in all others contaminants were added.

<sup>b</sup>Number of fatigue failures out of number of bearings tested.

<sup>c</sup>Probability expressed as a percentage that the fatigue life in the contaminated lubricant test series will be less than the life with the clean oil in test series I.

<sup>d</sup>Test series V was suspended after 448 test hours on each of the test bearings due to excessive bearing wear. No fatigue failures were encountered.

#### CONCLUDING REMARKS

This paper provides a broad survey of the lubrication of rolling-element bearings. It covers the basic functions of a lubricant, how to get it to the bearing and what happens if it is not successfully applied. Types of lubrication ranging from routine grease or splash methods to the more sophisticated under-race lubrication and cooling techniques are discussed.

Emphasis is on the state of the technology and limitations of the various methods of lubrication. For ball bearings, cylindrical roller bearings, and tapered-roller bearings, great gains have been made in the last decade to extend their speed capabilities. Under-race lubrication now allows reliable operation to such high speeds that a material and stress related problem (inner-race fracture) has become the limiting factor.

Temperature limitations of rolling-element bearings are less related to method of lubrication than to the temperature limitations of the lubricant itself. The use of under-race and outer-race cooling should however, extend the useful temperatures range by keeping raceways cooler for better EHD film conditions.

For very high temperatures, cryogenic temperatures or high vacuum environments, a designer now has some choice of solid film lubrication, transfer film lubrication, or lubrication with some of the newer greases. These are, however, limited life applications compared to proper oil lubrication.

The limits and techniques for jet lubrication are fairly well defined for ball bearings and tapered-roller bearings. Jet lubrication of cylindrical roller bearings has received little attention in recent years, but is being very satisfactorily used for both large and small size bearings in current aircraft turbine engines.

The use of energy storage flywheels, either in transportation systems or in stationary applications may require rolling-element bearings to operate reliably in a vacuum environment for extended periods of time. A wick lubrication system is proposed, but further testing is necessary to prove its long term operation.

The effects of EHD lubrication and the consequence of marginal EHD film thickness have become understood to the point that they can be used to better predict performance and life of rolling-element bearings and to aid in diagnosing lubrication-related bearing failures. However, the effects of chemical interactions, where there is appreciable asperity contact at low values, is an area requiring better understanding.

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### APPENDIX - SYMBOLS

a	semimajor axis of contact ellipse, m (in.)
b	semiminor axis of contact ellipse, m (in.)
E	modulus of elasticity, N/m <sup>2</sup> (psi)
E'	modified modulus of elasticity,
	$\frac{E}{(1 - \nu_1^2) + (1 - \nu_2^2)}$
e	Weibull slope
F	normal applied load, N (lb)
G	dimensionless material parameter, E'/P <sub>s</sub>
H <sub>min</sub>	dimensionless minimum film thickness, h/R <sub>x</sub>
h	film thickness, m (in.)
k	ellipticity parameter, a/b
P <sub>a</sub>	ambient pressure, N/m <sup>2</sup> (psi)
P <sub>s</sub>	asymptotic isoviscous pressure, N/m <sup>2</sup> (psi) (Ref. 3)
r	radius of curvature, m (in.)
R <sub>x</sub> , R <sub>y</sub>	effective radius of curvature, m (in.) $\frac{1}{R_x} = \frac{1}{r_{1x}} + \frac{1}{r_{2x}} ; \quad \frac{1}{R_y} = \frac{1}{r_{1y}} + \frac{1}{r_{2y}}$
U	dimensionless speed parameter, (u <sub>p</sub> <sub>d</sub> )/(E'R <sub>x</sub> )
u	surface velocity in x direction, m/sec (in./sec)
w <sub>p</sub>	dimensionless load parameter, F/(E'R <sub>x</sub> <sup>2</sup> )
A	lubricant film parameter, h/o
$\mu_a$	dynamic viscosity, N sec/m <sup>2</sup> (lb sec/in. <sup>2</sup> )
v	Poisson's ratio
$\sigma$	composite surface roughness, micrometers ( $\mu$ in.)
$\sigma_1, \sigma_2$	surface roughness of bodies 1 and 2, micrometers ( $\mu$ in.)
Subscripts:	
1, 2	bodies 1 and 2
x, y	coordinate directions